

DEPARTMENT OF THE INTERIOR
CANADA

HON. W. J. ROCHE, *Minister.* W. W. CORY, C.M.G., *Deputy Minister.*

PUBLICATIONS

OF THE

Dominion Observatory
OTTAWA

BIBLIOTHÈQUE
ÉCOLE POLYTECHNIQUE
DE MONTREAL
W. F. KING, C.M.G., LL.D., *Director.*

Vol. I, No. 10

Orbit of *b* Persei

BY

J. B. CANNON, M.A.

OTTAWA
GOVERNMENT PRINTING BUREAU
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ORBIT OF *b* PERSEI.

BY J. B. CANNON, M.A.

b Persei ($\alpha = 4^h 10^m 7^s$, $\delta = +50^\circ 03'$, mag. 4.5) was announced a binary by Prof. Campbell early in 1910* from measures of four plates taken, two in 1903, one in 1906 and one in 1909. Its spectrum is of A-type and a few of the plates obtained here show double lines. In the second spectrum thus appearing, the lines are not at all well defined and measures on them are only fair as will be seen by the magnitude of the residuals. The lines which show doubling are H_γ , H_δ , and in one case the iron line $\lambda 4045$.

Thirty-six plates dating from October 10, 1910, to April 24, 1911, were taken in all. The lines are about the average for this type. Being a double spectrum and the two components being separable in only a few plates, the measuring is more difficult than it would otherwise be. All plates were taken with the single-prism spectrograph, the length required for using the three prisms being too great for a star of this magnitude. Seed 27 plates were used throughout for the same reason, although a finer grained plate might have been very advantageous for showing doubling of lines.

The lines measured were:—

TABLE I.

Element.	Wave-Length.	Element.	Wave-Length.
Hydrogen.....	4861.527	Magnesium.....	4481.400
Hydrogen.....	4340.634	Iron.....	4549.766
Hydrogen.....	4101.890	Iron.....	4045.975
Hydrogen.....	3970.177 [†]	Calcium.....	3933.825

* L.O.B. V. 174. [†] Only on one or two plates.

Other iron lines appeared in a few of the plates but for the most part were not measurable.

Tables of observations follow. Table II contains the Lick observations and Table III the Ottawa observations. The phases are from corrected periastron, and the residuals from the corrected curve.

TABLE II.
LICK OBSERVATIONS.

Plate	Observer	Date	Exposure	Julian Day	Phase	COMPONENT I.			COMPONENT II.		
						Vel.	Wt.	O-C.	Vel.	Wt.	O-C.
		1903									
...	...	Nov. 3.....	...	2,416,422.024	1.209	+28.3	...	-12.5			
...	...	Nov. 16.....	...	435.989	1.428	-3.5	...	-0.5			
		1906									
...	...	Sept. 19.....	...	7,473.978	0.840	+49.5	...	-6.3			
		1909									
...	...	Sept. 2.....	...	8,552.017	0.591	+52.5		+12.5			
		1910									
...	...	Oct. 15.....	...	960.909	0.161	-31.4	...	-9.6			

TABLE III.
OTTAWA OBSERVATIONS.

Plate	Observer*	Date	Exposure	Julian Day	Phase	COMPONENT I.			COMPONENT II.		
						Vel.	Wt.	O-C.	Vel.	Wt.	O-C.
		1910	m.								
3728	P ¹	Oct. 10.....	60	2,418,955.830	1.191	+35.4	1.5	-7.6			
3738	C	Oct. 12.....	60	957.760	.067	-27.6	5.5	-1.6			
3809	P ¹	Nov. 9.....	72	985.630	.445	+12.8	3.7	-9.2			
3819	H	Dec. 5.....	70	9,011.720	.561	+41.8	2.5	+5.3			
3826	P ¹	Dec. 6.....	77	012.790	.114	-36.6	1.5	-9.5			
3854	P	Dec. 10.....	50	016.720	.989	+51.3	2.7	-5.2			

OTTAWA OBSERVATIONS—Continued.

Plate	Observer ^a	Date	Exposure ^b	Julian Day	Phase	COMPONENT I.			COMPONENT II.		
						Vel.	W.L.	O-C	Vel.	W.L.	O-C
		1910	m.								
3862	C-H	Dec. 12.....	60	2,419,018.730	1.97	— 8.9	1.0 + 4.1				
3870	P ^c	Dec. 15.....	70	021.620	1.297	+ 9.2	3.0 + 16.8				
3875	H	Dec. 16.....	70	022.640	.800	+50.7	2.2 + 3.3				
3885	P ^c	Dec. 21.....	68	027.540	1.118	+51.9	3.0 + 1.4	— 81.5	.2	— 5.0	
3907	H	Dec. 30.....	65	030.715	1.129	+50.7	3.2 + 9.7	— 70.4	.7	+ 1.6	
		1911									
3919	C	Jan. 5.....	60	042.719	1.024	+51.0	4.5 + 5.5				
3931	P	Jan. 12.....	110	049.680	.349	+ 4.4	2.7 + 2.6				
3939	C-II	Jan. 16.....	74	053.724	1.338	+24.2	4.7 + 6.7				
3949	H	Jan. 17.....	60	054.792	.878	+59.3	3.2 + 2.8				
3960	P ^c	Jan. 18.....	112	055.666	.225	—10.0	.5 + 3.0				
3963	P	Jan. 19.....	60	056.642	1.201	+48.4	4.5 + 6.4				
3996	P ^c	Feb. 15.....	65	083.610	.678	+43.3	5.0 + 3.7	— 78.7	.2	— 16.7	
4005	C	Feb. 20.....	68	088.530	1.016	+59.0	4.2 + 2.5				
4017	H	Feb. 25.....	61	093.510	1.414	— 3.3	3.0 + 2.8				
4018	C	Feb. 27.....	70	095.490	.340	+ 5.8	3.7 + 0.0				
4031	H	Feb. 28.....	60	096.530	1.379	+14.1	3.0 + 2.0				
4033	P ^c	Mar. 1.....	87	097.643	.965	+60.2	3.2 + 3.6				
4034	P	Mar. 2.....	45	098.491	.286	+ 3.1	3.2 + 6.6				
4057	C	Mar. 6.....	68	102.549	1.289	+20.8	3.7 + 6.7				
4068	H	Mar. 7.....	69	103.580	.793	+45.3	4.2 + 3.7	— 119.0	.2	— 33.0	
4078	P ^c	Mar. 8.....	75	104.583	.269	—14.3	3.5 + 8.3				
4087	C	Mar. 10.....	68	106.542	.700	+55.6	3.5 + 6.6				
4100	C	Mar. 13.....	75	109.560	.664	+45.4	2.5 + 0.6				
4112	H	Mar. 14.....	70	110.550	.127	—22.7	3.5 + 2.5	+168.8	.6	—28.2	
4135	C	Mar. 24.....	70	120.542	.955	+58.7	2.5 + 2.0	— 77.3	.2	+22.7	
4143	H	Mar. 28.....	77	124.638	.489	+33.5	2.2 + 5.5				
4176	C	April 10.....	70	137.542	1.154	+47.8	2.7 + 0.2				
4185	H	April 11.....	70	138.536	.621	+34.4	3.0 + 7.6				
4195	H	April 14.....	75	141.550	.580	+42.1	2.7 + 4.9				
4246	C	April 24.....	70	151.542	1.409	+ 3.5	2.7 + 2.5				

^aP=Plaskett;^bH=Harper;^cP^c=Parker;^dC=Cannon.

MEASURES OF *b* PERSEL

λ	3728		3738p.*		3738s.*		3809		3819		3826		3854		
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	
4861-527	+ 7.54	1					+ 23.43	1	+ 60.71	1			+ 69.90	1	
4549-766	+ 22.24	1	- 39.75	1	+ 100.98	1			+ 33.08	1			+ 59.87	1	
4534-139													+ 62.99	1	
4181-400	+ 14.91	1	- 55.16	1			+ 13.63	1	+ 50.62	1	- 34.15	1		+ 35.55	1
4395-286															
4352-006	+ 7.80	1											+ 47.44	1	
4340-634			- 49.29	1	+ 124.27	1	- 0.81	1	+ 30.41	1					
4101-890			- 47.91	1					+ 41.79	1					
3970-177							+ 0.96	1							
3933-825			- 44.62	1			- 14.13	1	+ 37.11	1					
Weighted mean	+ 14.93		- 47.53		+ 112.62		+ 2.84		+ 43.72		- 34.15		+ 55.50		
V_a	+ 20.70		+ 20.12		+ 20.12		+ 10.14		- 1.56		- 2.06		- 3.87		
V_d	+ .01		+ .09		+ .09		+ .14		- .05		- .14		- .07		
Curv.	- .28		- .28		- .28		- .28		- .28		- .28		- .28		
Radial Velocity	+ 35.4		- 27.6		+ 132.6		+ 12.8		+ 41.8		- 36.6		+ 51.3		

* p. = primary.
s. = secondary.

MEASURES OF *b* PERSEI—Continued.

λ	3862		3870		3875		3885p.		3885p.		3885s.		3907p.		
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	
4861.527	+ 24.73	1	+ 67.61	1	+ 49.59	1	+ 33.00	1	
4549.766	+ 17.61	1	+ 66.40	1	
4534.139	
4481.400	— 5.86	1	+ 5.36	1	+ 69.54	1	+ 73.31	1	+ 73.57	1	+ 64.18	1	
4340.634	— 4.51	1	+ 9.14	1	+ 58.89	1	+ 81.45	1	+ 82.61	1	— 72.43	1	+ 80.99	1	
4308.081	— 0.56	1	
4128.211	— 4.03	1	
4101.890	+ 33.90	1	+ 37.65	1	+ 88.98	1	
3933.825	+ 16.06	1	+ 62.65	1	+ 45.26	1	+ 50.84	1	— 77.46	1
Weighted mean	— 3.74	+ 15.51	+ 57.50	+ 57.71	+ 66.51	— 72.43	+ 72.76	
V_a	— 4.81	— 6.10	— 6.57	— 8.76	— 8.76	— 8.76	— 12.67	
V_d	— .09	+ .05	+ .01	— .05	— .05	— .05	— .12	
Curv.	— .28	— .28	— .28	— .28	— .28	— .28	— .28	
Radial Velocity	— 8.9	+ 9.2	+ 50.7	+ 48.6	+ 57.4	— 81.5	+ 59.7	

MEASURES OF *b* PERSEI—Continued.

λ	3907s.		3919		3931		3939		3949		3960		3963		
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	
4861-527	+	85.12	1	+	16.38	1	+	53.64	1	+	123.98	1	+	97.32	1
4549-766							+	44.89	1	+	100.58	1			
4481-400	+	64.06	2	+	19.40	1	+	43.24	1	+	77.42	1	+	9.82	1
4352-096	+	66.87	1						+	65.12	1				
4340-634	—	57.04	1	+	99.50	1	+	28.46	1	+	42.58	1	+	53.22	1
4101-890	—	58.07	1	+	90.23	1							+	71.50	1
3968-625							+	40.29	1				+	79.16	1
3933-825			+	47.67	1			+	43.51	1			+	53.69	1
													+	63.56	1
Weighted mean	—	57.38		+	66.49		+	22.42		+	43.66		+	9.82	
V_a	—	12.67		—	15.06		—	17.63		—	18.98		—	19.39	
V_d	—	.12		—	.10		—	.12		—	.18		—	.21	
Curv.	—	.28		—	.28		—	.28		—	.28		—	.28	
Radial Velocity	—	70.4		+	51.0		+	4.4		+	24.2		+	59.3	
													—	10.0	
													+	48.4	

MEASURES OF δ PERSEI—Continued.

λ	3996p.	3996a.	4005	4017	4018	4031	4033							
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.		
4861-527	+ 58.94	1	+ 73.39	1	+ 28.75	1	+ 31.96	1	+ 52.52	1	+ 71.31	1
4549-766	+ 75.24	1	+ 84.57	1	+ 12.27	1
4534-139	+ 61.16	1
4515-508	+ 85.93	1	+ 86.00	1	+ 29.20	1	+ 27.94	1	+ 56.02	1	+ 98.51	1
4481-400	+ 61.84	1
4352-006	+ 47.50	1	+ 27.31	1
4340-634	+ 77.75	1	+ 94.41	1	- 11.11	1	+ 47.55	1	+ 23.60	1	+ 84.69	1
4325-939	+ 15.69	1
4315-178	+ 15.34	1
4101-890	+ 81.76	1	+ 57.20	1	+ 90.71	1
4071-901	+ 62.70	1	+ 19.46	1
4063-756	+ 75.11	1	+ 36.85	1
4045-975	+ 60.01	1	- 51.65	1
3933-825	+ 54.83	1	+ 94.93	1	+ 19.64	1	+ 35.53	1	+ 28.79	1	+ 83.87	1
Weighted mean	+ 69.36	1	- 51.05	1	+ 85.45	1	+ 23.48	1	+ 22.56	1	+ 40.95	1	+ 87.10	1
V_x	- 25.06	1	- 25.66	1	- 26.12	1	- 26.39	1	- 26.44	1	- 26.46	1	- 26.45	1
V_d	- .14	1	- .14	1	- .04	1	- .09	1	- .05	1	- .10	1	- .19	1
Curv.	- .28	1	- .28	1	- .28	1	- .28	1	- .28	1	- .28	1	- .28	1
Radial Velocity	+ 43.3	1	- 78.7	1	+ 50.0	1	- 3.3	1	+ 5.8	1	+ 14.1	1	+ 60.2	1

MANUFACTURE OF PLASTIC CLOTHES

MEASURES OF *b* PERSEL—Continued.

λ	4112p.		4112a.		4135p.		4135a.		4143		4176		4185	
	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.	Vel.	Wt.
4861-527	+ 12.99	1			+ 108.24	1			+ 55.09	1	+ 78.11	1	+ 41.51	1
4572-156									+ 52.09	1				
4549-766									+ 84.98	1			+ 83.78	1
4481-400	+ 9.56	1			+ 67.63	1			+ 47.47	1	+ 74.53	1	+ 64.34	1
4395-286									+ 39.10	1				
4383-720									+ 80.77	1				
4340-634	+ 1.97	1			+ 92.10	1	+ 52.41	1	+ 69.07	1	+ 61.27	1	+ 36.18	1
4325-939									+ 43.51	1				
4101-890	+ 24.43	1	+ 195.00	1	+ 62.02	1					+ 65.61	1	+ 59.36	1
3933-825	+ 15.37	1			+ 82.45	1					+ 62.91	1	+ 44.04	1
Weighted mean	+ 3.58		+ 195.00		+ 83.57		+ 52.41		+ 57.61		+ 68.70		+ 54.98	
V_a	+ 25.79		+ 25.79		+ 24.40		+ 24.40		+ 23.61		+ 20.44		+ 20.15	
V_t	+ .16		+ .16		+ .18		+ .18		+ .19		+ .19		+ .19	
Curv.	+ .28		+ .28		+ .28		+ .28		+ .28		+ .28		+ .28	
Radial Velocity	+ 22.6		+ 168.8		+ 58.7		+ 77.3		+ 33.5		+ 47.8		+ 34.4	

MEASURES OF A PERSONAL STATE

The observations were combined into fourteen groups and the resulting normal places used in obtaining, by means of Dr. King's graphic method, preliminary elements for the orbit. The normal places with mean velocities, mean phases, weights and residuals from final curve are given below.

NORMAL PLACES

	Julian Day.	Phase	Velocity.	Weight.	Residual
1	2,419,045.213	.464	+21.7	1.0	-2.21
2	100.036	.500	+39.6	2.0	+1.04
3	099.214	.682	+48.3	2.0	+1.66
4	068.469	.810	+49.4	1.5	-4.04
5	073.649	.961	+56.7	2.0	-0.10
6	069.281	1.020	+55.4	1.0	-0.37
7	078.533	1.137	+52.5	2.0	+3.41
8	018.838	1.197	+43.5	1.0	+0.77
9	064.280	1.312	+19.8	2.0	-3.93
10	116.991	1.399	+6.0	1.5	+1.88
11	018.730	1.472	-8.9	.5	+3.52
12	017.472	.100	-27.3	1.5	-0.83
13	085.016	.252	-12.6	.5	-3.81
14	082.780	.318	+4.1	1.5	+2.11

The preliminary elements obtained were:—

$$P = 1.52732 \text{ days}$$

$$e = .25$$

$$\omega = 150^\circ$$

$$K = 43.5 \text{ km.}$$

$$\gamma = 21.92 \text{ km.}$$

$$T = 2,418,956.145 \text{ J. D.}$$

To get a closer approximation to the true values of the elements a least-squares solution was applied. P was assumed as determined but e , ω , K , γ and T were used in the solution.

Observation equations were formed as follows:—

<i>x</i>	<i>y</i>	<i>z</i>	<i>u</i>	<i>v</i>	<i>-n</i>	Weight.
1	+ .072	+ .405	+ .832	- .640	+3.350	1.0
1	+ .390	- .287	+ .670	- .471	-0.730	2.0
1	+ .562	- .685	+ .503	- .356	-1.940	2.0
1	+ .722	- .927	+ .219	- .196	+3.950	1.5
1	+ .782	- .811	- .178	+ .033	-0.760	2.0
1	+ .758	- .571	- .348	+ .149	-0.500	1.0
1	+ .602	+ .185	- .690	+ .450	-4.380	2.0
1	+ .450	+ .652	- .871	+ .672	-2.020	1.0
1	- .024	+1.195	-1.106	+1.147	+1.080	2.0
1	- .526	+ .617	-1.076	+1.337	-6.050	1.5
1	- .938	- .469	- .818	+1.070	-9.970	.5
1	-1.173	- .374	+ .166	- .399	-1.820	1.5
1	- .671	+1.019	+ .765	- .868	+5.310	.5
1	- .416	+1.037	+ .855	- .831	-0.270	1.5

where $x = \delta\gamma$

$y = \delta K$

$z = K\delta e$

$u = K\delta\omega$

$v = \frac{K\mu\delta T}{(1-e^2)^{\frac{3}{2}}}$

Normal equations resulting from these observation equations were as follows:—

$$\begin{aligned}
 20x + 3.010y + .483z - 1.786u + 1.772v - 22.595 &= 0 \\
 + 7.852y - 3.808z - .187u - .105v + 5.719 &= 0 \\
 10.569z - 3.200u + 3.458v - .676 &= 0 \\
 10.036u - 9.506v + 23.982 &= 0 \\
 9.597v - 24.408 &= 0
 \end{aligned}$$

Solving these equations the values for the corrections were found to be:—

$$\delta\gamma = + 1.20 \text{ km.}$$

$$\delta K = - 1.925 \text{ km.}$$

$$\delta e = - .038$$

$$\delta\omega = +0^{\circ}.85$$

$$\delta T = + 0.018 \text{ day}$$

The value of Σprv was greatly reduced being brought down from 241 to 128. Disagreement between computed and observation equation

residuals led to a second solution being made. Observation equations, determined from the corrected values of the elements using the same substitutions as before, are as follows:—

<i>x</i>	<i>y</i>	<i>z</i>	<i>u</i>	<i>v</i>	$-\pi$	<i>p</i>
1	+ .012	+ .579	+ .877	- .729	+1.940	1.0
1	+ .368	- .149	+ .730	- .549	-1.190	2.0
1	+ .563	- .607	+ .560	- .417	-1.760	2.0
1	+ .746	- .965	+ .262	- .228	+4.730	1.5
1	+ .813	- .817	- .158	+ .037	+0.230	2.0
1	+ .787	- .570	- .336	+ .166	+0.460	1.0
1	+ .624	+ .194	- .601	+ .488	-3.450	2.0
1	+ .407	+ .649	- .861	+ .695	-0.960	1.0
1	+ .006	+1.144	-1.085	+1.115	+3.580	2.0
1	- .461	+ .653	-1.065	+1.272	-2.050	1.5
1	- .849	- .304	- .851	+1.077	-3.270	.5
1	-1.175	- .679	+ .038	- .193	+1.500	1.5
1	- .766	+ .841	+ .711	- .853	+3.880	.5
1	- .513	+1.039	+ .841	- .876	-2.300	1.5

The consequent normal equations were:—

$$\begin{aligned}
 20x + 3.102y + .528z - 1.565u + 1.555v - .525 &= 0 \\
 8.116y - 3.332z - .215u + .005v - 1.174 &= 0 \\
 10.635z - 2.973u + 3.172v - 2.693 &= 0 \\
 10.157u - 9.667v + .683 &= 0 \\
 9.612v - .970 &= 0
 \end{aligned}$$

The solution of these equations gave the second corrections —

$$\begin{aligned}
 \delta\gamma &= - .03 \text{ km.} \\
 \delta K &= + .31 \text{ km.} \\
 \delta e &= + .008 \\
 \delta\omega &= + 0^\circ.90 \\
 \delta T &= + 0.0035 \text{ day}
 \end{aligned}$$

The new values for the elements gave satisfactory comparison between computed and observation residuals, and a second lowering of Σp_{vv} from 128 to 126.

The probable error of a normal place of weight unity was computed and found to be ± 2.5 , while that of a plate of average weight was ± 3.9 .

The probable errors of the elements were also computed, and are given below after the final values of each.

The following table gives a summary of the values of the elements preliminary, first corrected and final:—

Element.	Preliminary.	First Corrected.	Final.
P	1.52732 days	1.52732 days	1.52732 days
e	.25	.212	.22
ω	159°	159°.85	151°.75
K	43.5 km.	41.575 km.	41.89 km.
γ	21.92 km.	23.12 km.	23.09 km.
T	2,418,956.145 J. D.	2,418,956.163 J. D.	2,418,956.166 J. D.
$a \sin i$			837,000 km.

Coming to the secondary, only six plates showed lines measurable. No attempt was made to get elements independently of the primary, the number of plates not being considered sufficient for that. The values of γ and T finally accepted for the primary were taken for the secondary, ω_1 taken $180^\circ + \omega$ and the K was obtained by trial graphically. Those given probably suit the six observations as well as any. The elements are:—

$$\begin{aligned}
 P_1 &= 1.52732 \text{ days} \\
 e_1 &= .22 \\
 \omega_1 &= 331^\circ.75 \\
 K_1 &= 152.5 \text{ km.} \\
 \gamma_1 &= 23.09 \text{ km.} \\
 T_1 &= 2,418,956.166 \text{ J. D.} \\
 a_1 \sin i_1 &= 3,048,000 \text{ km.}
 \end{aligned}$$

The relation existing between the masses of the two bodies is given by $M : M_1 = K_1 : K = 152.5 : 42$ or $3.6 : 1$.

The figure gives the two curves. The double circles are the observations on secondary component and the triple circles are Lick observations.

Dominion Observatory,
Ottawa,
June, 1911.

